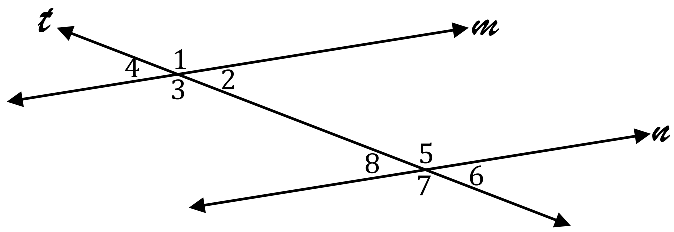
Properties of Parallel Lines

A transversal is a line that intersects two coplanar lines at two distinct points.



Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal. $\angle 3$ & $\angle 5$ and $\angle 2$ & $\angle 8$ are alternate interior angles.

Same-side interior angles are angles that lie on the same side of the transversal t and between m and n. $\angle 2$ & $\angle 5$ and $\angle 3$ & $\angle 8$ are same-side interior angles.

Corresponding angles lie on the same side of the transversal t and in corresponding positions relative to m and n. $\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 4$ & $\angle 8$ and $\angle 3$ & $\angle 7$ are corresponding angles.

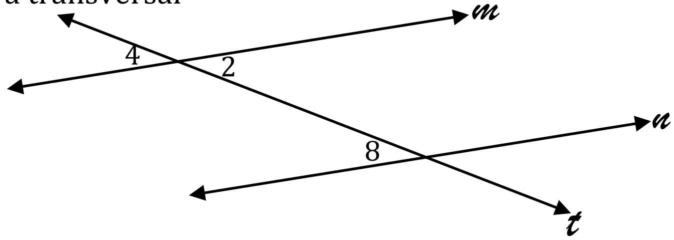
Corresponding Angles Postulate (CAP)
If a transversal intersects two parallel lines, then corresponding angles are congruent. $\angle 1 \cong \angle 5$; $\angle 2 \cong \angle 6$; $\angle 4 \cong \angle 8$; and $\angle 3 \cong \angle 7$

Alternate Interior Angles Theorem (AIAT)
If a transversal intersects two parallel lines, then corresponding angles are congruent.
$\angle 3 \cong \angle 5$ and $\angle 2 \cong \angle 8$

Same-Side Interior Angles Theorem (SSIAT)
If a transversal intersects two parallel lines, then same-side angles are supplementary.
$m\angle 2 + m\angle 5 = 180$ and $m\angle 3 + m\angle 8 = 180$

Proof of Alternate Interior Angles Theorem (AIAT)

Given: $m \parallel n$ and t is a transversal

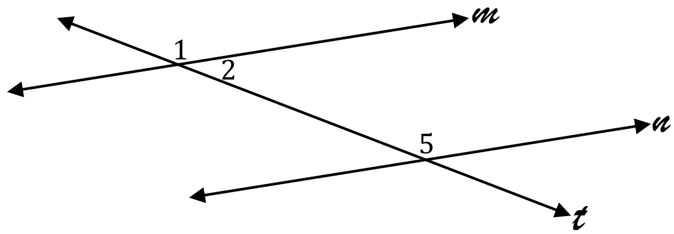


Prove: $\angle 2 \cong \angle 8$

Steps	Reasons
1. $m \parallel n$	Given
2. $\angle 2 \cong \angle 4$	Vertical Angles Theorem (VAT)
3. $\angle 4 \cong \angle 8$	Corresponding Angles Postulate (CAP)
4. $\angle 2 \cong \angle 8$	Transitive Property of Congruence

Proof of Same-Side Interior Angles Theorem (SSIAT)

Given: $m \parallel n$ and t is a transversal

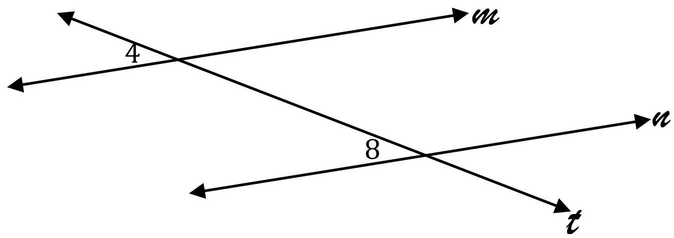


Prove: $\angle 2$ and $\angle 5$ are supplementary

Steps	Reasons
1. $m \parallel n$	Given
2. $\angle 1 \cong \angle 5$	Corresponding Angles Postulate (CAP)
3. $m\angle 1 = m\angle 5$	Definition of Congruence
4. $\angle 1$ and $\angle 2$ are adjacent supplementary angles	Given
5. $m\angle 1 + m\angle 2 = 180$	Definition of Supplementary Angles
6. $m\angle 5 + m\angle 2 = 180$	Substitution
7. $\angle 2$ and $\angle 5$ are supplementary angles	Definition of Supplementary Angles

Proving Lines Parallel
(In this case, we will be given two lines cut by a transversal and we will be proving that the two lines are parallel)

Converse of Corresponding Angles Postulate (CCAP)
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

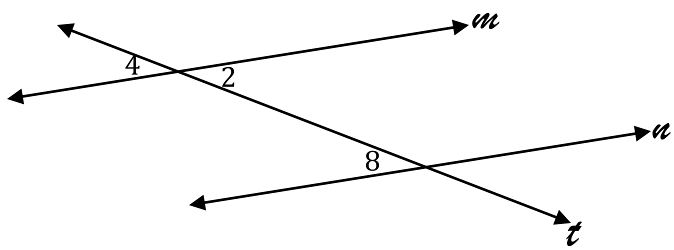


$m // n$ since the corresponding angles 4 and 8 are congruent.

Converse of Alternate Interior Angles Theorem (CAIAT)
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

Proof of Converse of Alternate Interior Angles Theorem (AIAT)

Given: lines m and n are cut by transversal t and $\angle 2 \cong \angle 8$

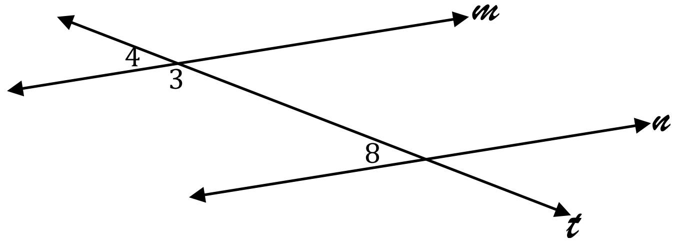


Prove: $m // n$

Steps	Reasons
1. lines m and n are cut by transversal t and $\angle 2 \cong \angle 8$	Given
2. $\angle 4 \cong \angle 2$	Vertical Angles Theorem (VAT)
3. $\angle 4 \cong \angle 8$	Transitive Property of Congruence
4. $m // n$	Converse of Corresponding Angles Postulate (CCAP)

Converse of Same-Side Interior Angles Theorem (CSSIAT)
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

Proof of Converse of Side Interior Angles Theorem (CSSIAT)
Given: lines m and n are cut by transversal t and $\angle 3$ and $\angle 8$ are supplementary

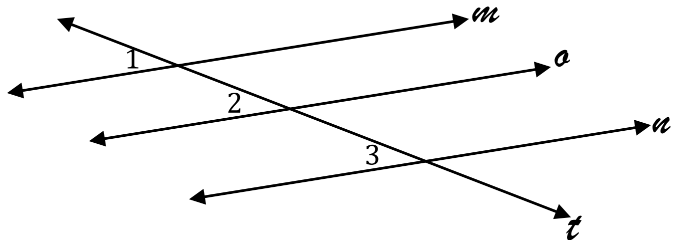


Prove: $m // n$

Steps	Reasons
1. lines m and n are cut by transversal t and $\angle 3$ and $\angle 8$ are supplementary	Given
2. $m\angle 3 + m\angle 8 = 180$	Definition of Supplementary Angles
3. $\angle 3$ and $\angle 4$ are adjacent supplementary angles	Given
4. $m\angle 3 + m\angle 4 = 180$	Definition of Supplementary Angles
5. $m\angle 3 + m\angle 8 = m\angle 3 + m\angle 4$	Substitution/Transitive Property
6. $m\angle 8 = m\angle 4$	Subtraction Property of Equality (SPE)
7. $\angle 8 \cong \angle 4$	Definition of Congruence
8. $m // n$	Converse of Corresponding Angles Postulate (CCAP)

Theorem:
If two lines are parallel to the same line, then they are parallel to each other.

Proof :
Given: $m // o$ and $n // o$ and a transversal t

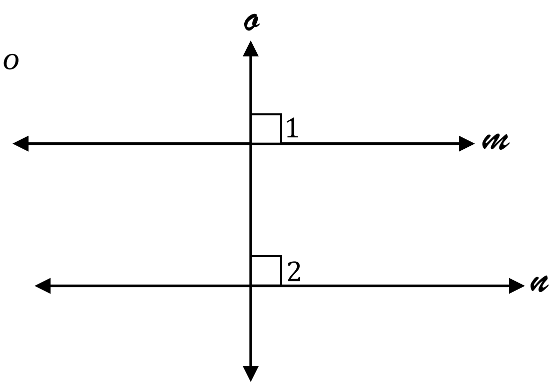


Prove: $m // n$

Steps	Reasons
1. $m // o$ and $n // o$ and a transversal t	Given
2. $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$	Corresponding Angles Postulate (CAP)
3. $\angle 1 \cong \angle 3$	Transitive Property
4. $m // n$	Converse of Corresponding Angles Postulate (CCAP)

Theorem: In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Proof :
Given: $m \perp o$ and $n \perp o$



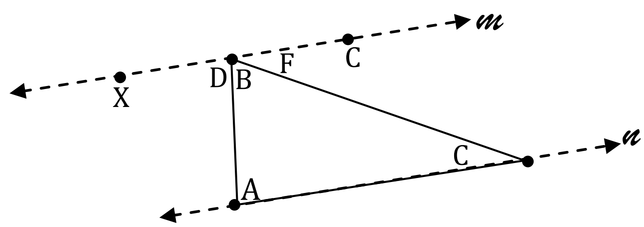
Prove: $m // n$

Steps	Reasons
1. $m \perp o$ and $n \perp o$	Given
2. $\angle 1$ and $\angle 2$ are right angles	Definition of Perpendicular Lines
3. $\angle 1 \cong \angle 2$	All right angles are congruent
4. $m // n$	Converse of Corresponding Angles Postulate (CCAP)

Parallel Lines and the Triangle Angle-Sum Theorem

Triangle Angle-Sum Theorem (TAST)
The sum of the measures of the angles of a triangle is 180.

Proof of Triangle Angle-Sum Theorem (TAST)
Given: triangle ABC



Prove: $m\angle A + m\angle B + m\angle C = 180$

Steps	Reasons
1. triangle ABC	Given
2. construct parallel lines m and n containing side AC and vertex B	Construction (Given a line and a point not on the line, it is possible to construct the line containing the points and parallel to the given line)
3. $\angle A \cong \angle D$; $\angle C \cong \angle F$	Alternate Interior Angles Theorem (AIAT)
4. $m\angle A = m\angle D$; $m\angle C = m\angle F$	Definition of Congruence
5. $\angle XBY$ is a straight angle	Given
6. $m\angle XBY = 180$	Definition of a Straight Angle
7. $m\angle D + m\angle B + m\angle F = m\angle XBY$	Angle Addition Postulate
8. $m\angle A + m\angle B + m\angle C = 180$	Substitution

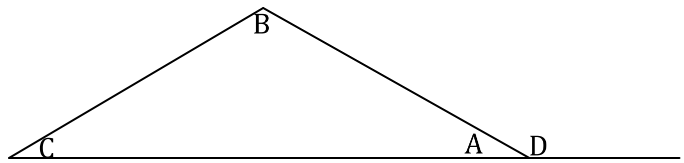
Classifications of Triangles According to Angles

1. ***Equiangular*** **–** all angles are congruent
2. ***Acute*** **–** all angles are acute
3. ***Right*** **–** one right angle
4. ***Obtuse*** **–** one obtuse angle

Classification of Triangles According to Sides
1. ***Equilateral*** – all sides are congruent
2. ***Isosceles*** – at least two sides congruent
3. ***Scalene*** – n0 sides congruent

An ***exterior angle*** of a polygon is an angle formed by a side and an extension of an adjacent side.

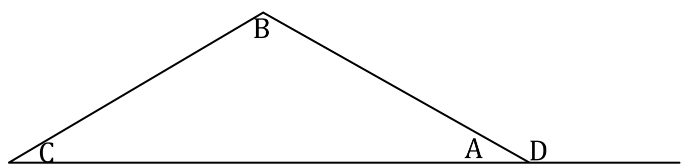
For each exterior angle of a triangle, the ***remote interior angles*** are the two non adjacent interior angles.



In the figure above, $\angle D$ is an exterior angle and its two remote interior angles are $\angle B$ and $\angle C$.

Triangle Exterior Angle Theorem (TEAT)
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

Proof of Triangle Exterior Angle Theorem (TEAT)
Given: Triangle ABC with exterior angle D



Prove: $m\angle D = m\angle B + m\angle C$

Steps	Reasons
1. triangle ABC with exterior angle D	Given
2. $\angle A$ and $\angle D$ are adjacent supplementary angles	Given
3. $m\angle A + m\angle D = 180$	Definition of Supplementary Angles
4. $m\angle A + m\angle B + m\angle C = 180$	Triangle Angle Sum Theorem (TAST)
5. $m\angle A + m\angle D = m\angle A + m\angle B + m\angle C$	Substitution/Transitive Property
6. $m\angle D = m\angle B + m\angle C$	Subtraction Property of Equality (SPE)

The Polygon Angle-Sum Theorem

A ***polygon*** is a closed plane figure with at least three sides that are segments. The sides intersect only at its endpoints and no adjacent sides are collinear.

Polygons can be classified according to the number of its sides.

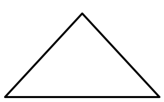
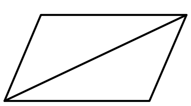
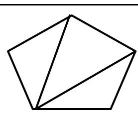
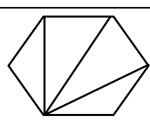
Number of Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon/Septagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon
n	n-gon

Polygons can be classified as convex or concave.

1. ***Concave*** **–** if the polygon has no diagonal with points lying outside the polygon
2. ***Concave*** – if the polygon has at least one diagonal with points lying outside the polygon.

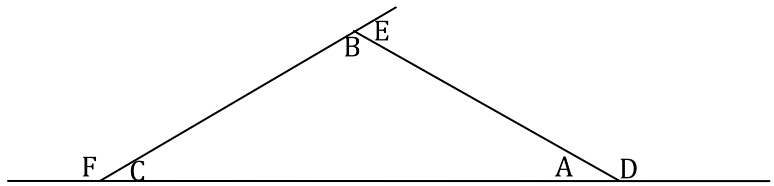
Polygon Angle-Sum Theorem (PAST)
The sum of the measures of the angles of a an n-gon is (n-2)180.

Proof Polygon Angle-Sum Theorem (PAST): (By Mathematical Induction)

	Number of Sides	Number of Triangles Formed	Sum of the measure of Angles
	3	1	180 = 1(180)
	4	2	360 = 2(180)
	5	3	540 = 3(180)
	6	4	720 = 4(180)
…	…	…	…
	n	n-2	(n-2)180

Polygon Exterior Angle-Sum Theorem(PEAST)
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

Proof of Polygon Exterior Angle-Sum Theorem(PEAST)
Let us consider a triangle,
Given: Triangle ABC with exterior angles D, E and F



Prove: $m\angle D + m\angle E + m\angle F = 360$

Steps	Reasons
1. Triangle ABC with exterior angles D, E and F	Given
2. $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$ are adjacent supplementary angles	Given
3. $m\angle A + m\angle D = 180$	Definition of Supplementary Angles
4. $m\angle B + m\angle E = 180$	Definition of Supplementary Angles
5. $m\angle C + m\angle F = 180$	Definition of Supplementary Angles
6. $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F = 180 + 180 + 180$	Addition Property of Equality (APE)
7. $m\angle A + m\angle B + m\angle C = 180$	Triangle Angle-Sum Theorem (TAST)
8. $180 + m\angle D + m\angle E + m\angle F = 180 + 180 + 180$	Substitution
9. $m\angle D + m\angle E + m\angle F = 360$	Subtraction Property of Equality (SPE)

This is true for any polygon.

Lines in the Coordinate Plane

The following are the forms of a line:

Equation of a line		
Slope-intercept form	$y = mx + b$	m is the slope b is the y-intercept
Point-Slope form	$y - y_1 = m(x - x_1)$	m is the slope (x_1, y_1) is a given point
Two-Point Form	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$	$\frac{y_2 - y_1}{x_2 - x_1}$ is the slope (x_1, y_1) is a given point
Standard form	$Ax + By = C$	$-\frac{A}{B}$ is the slope $\frac{C}{B}$ is the y-intercept
General Form	$Ax + By + C = 0$	$-\frac{A}{B}$ is the slope $-\frac{C}{B}$ is the y-intercept

Two lines are said to be parallel if the slopes of the line are equal but their y-intercepts are not equal. That is, given $y = m_1x + b_1$ and $y = m_2x + b_2$, if $m_1 = m_2$ and $b_1 \neq b_2$, then the two lines are parallel.

Two lines are said to be perpendicular of the products of their slopes is -1. That is, given $y = m_1x + b_1$ and $y = m_2x + b_2$, if $m_1 \cdot m_2 = -1$, then the two lines are parallel.

Constructing Parallel and Perpendicular Lines

Constructing Parallel Lines given a line and a point not on the line.

1. Draw a line that will contain the given point and will intersect the given line. (An angle will be formed by these two lines)
2. Using the compass, construct an arc that will intersect the line formed in #1 and the given line. Mark these points of intersection.
3. Using the same compass measurement in #2, construct an arc where the compass end will be positioned at the given point.
4. Adjust the compass opening in order to measure the points of intersection obtained from #2.
5. Using the same compass opening, draw an arc that will intersect the arc obtained in #4. Mark the point of intersection.
6. Connect the result of the point obtained in #5 and connect it with the given line.

Constructing Perpendicular Lines Given a Line and a Point on the Line

1. Place the compass end at the given point. Choose a compass opening and draw arcs on both sides of the line. Mark the points of intersection.
2. Choose a compass opening and construct arcs by placing the compass end at the points that resulted from #1. Mark the point of intersection.
3. Connect the result of #2 and the given point.

Constructing Perpendicular Lines Given a Line and a Point not on the Line

1. Place the compass end at the given point. Choose a compass opening and construct an arc that will intersect the given line twice. Mark the points of intersection.
2. Choose a compass opening and construct arcs by placing the compass end at the points that resulted from #1. Mark the point of intersection.
3. Connect the result of # 2 and the given point.