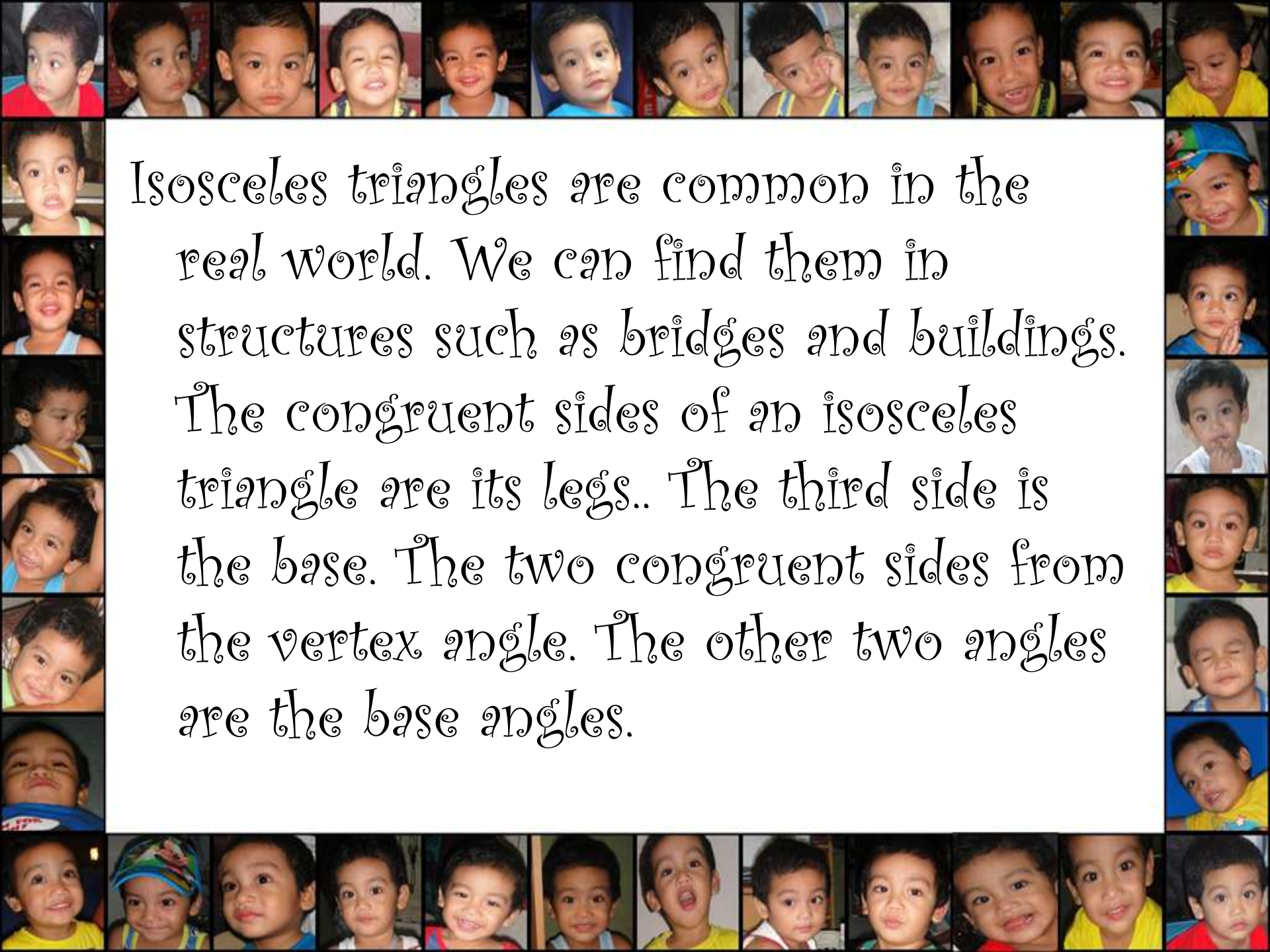
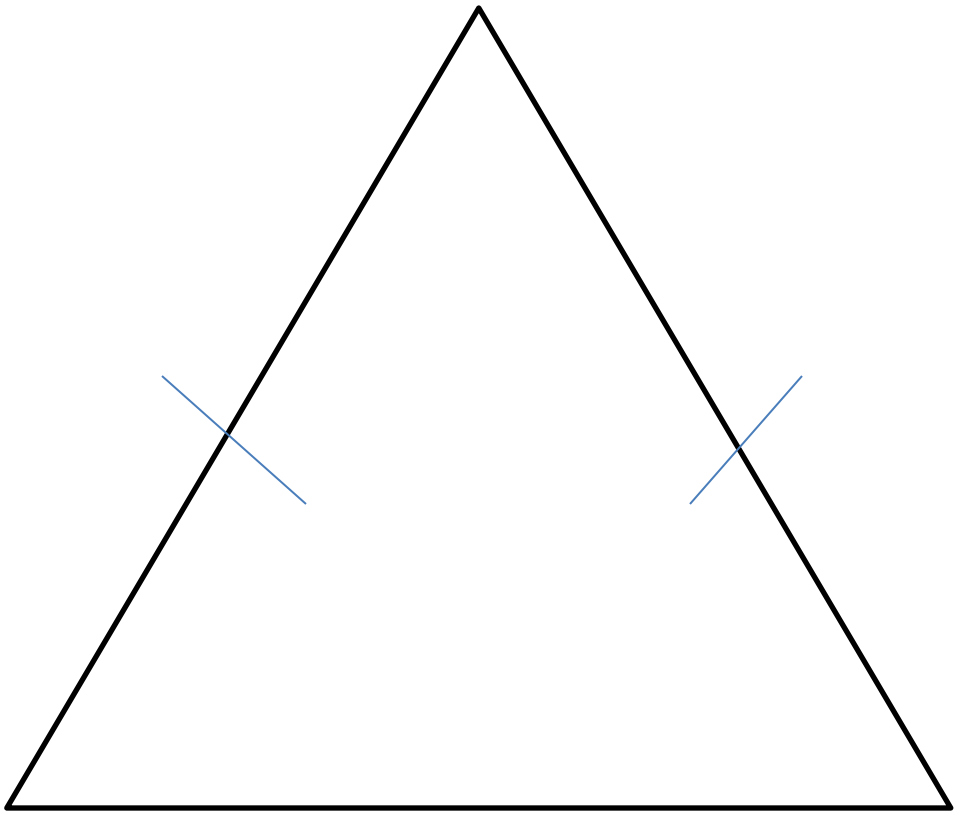
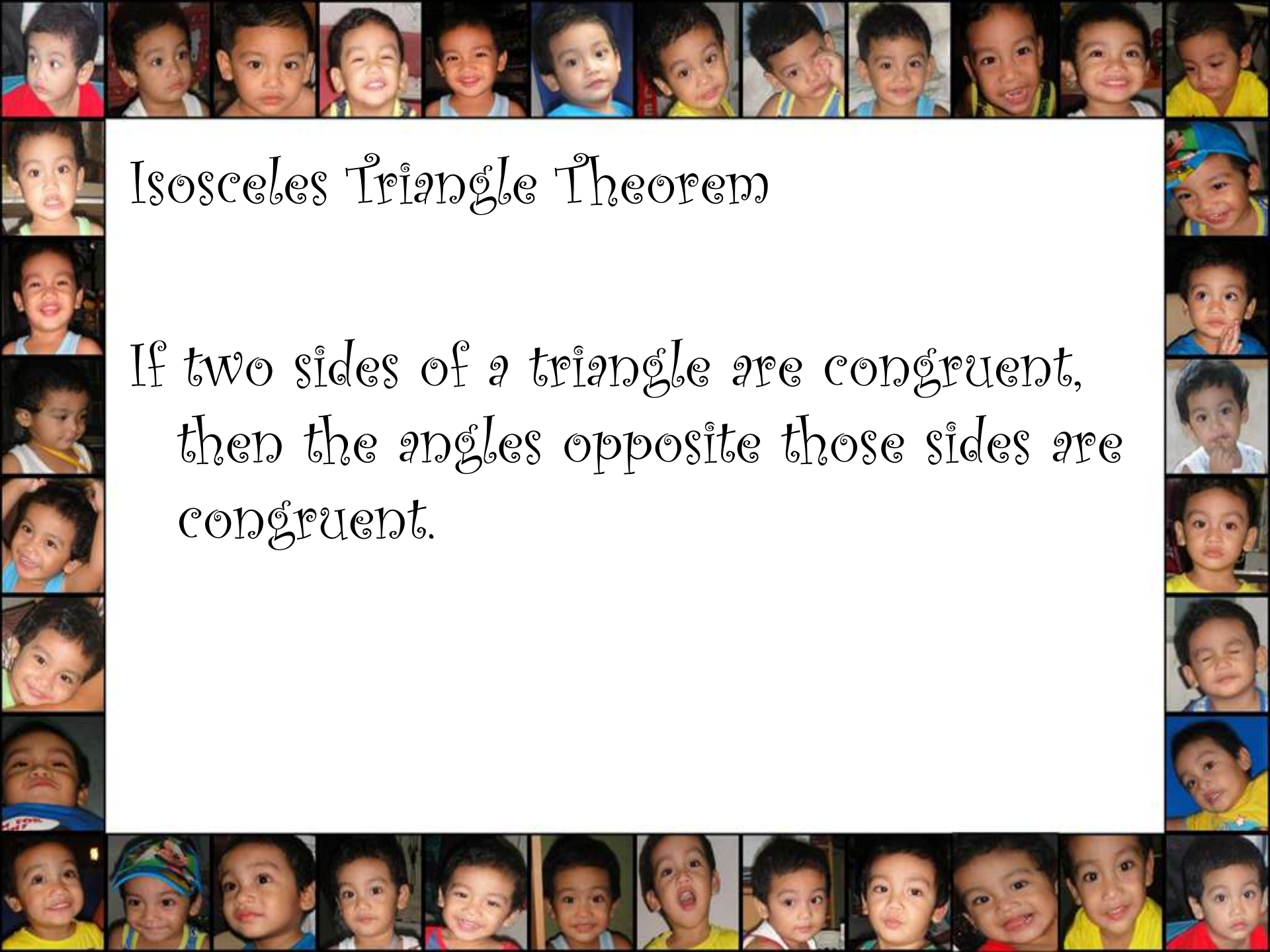
Isosceles and Equilateral Triangles



Isosceles triangles are common in the real world. We can find them in structures such as bridges and buildings. The congruent sides of an isosceles triangle are its legs.. The third side is the base. The two congruent sides from the vertex angle. The other two angles are the base angles.



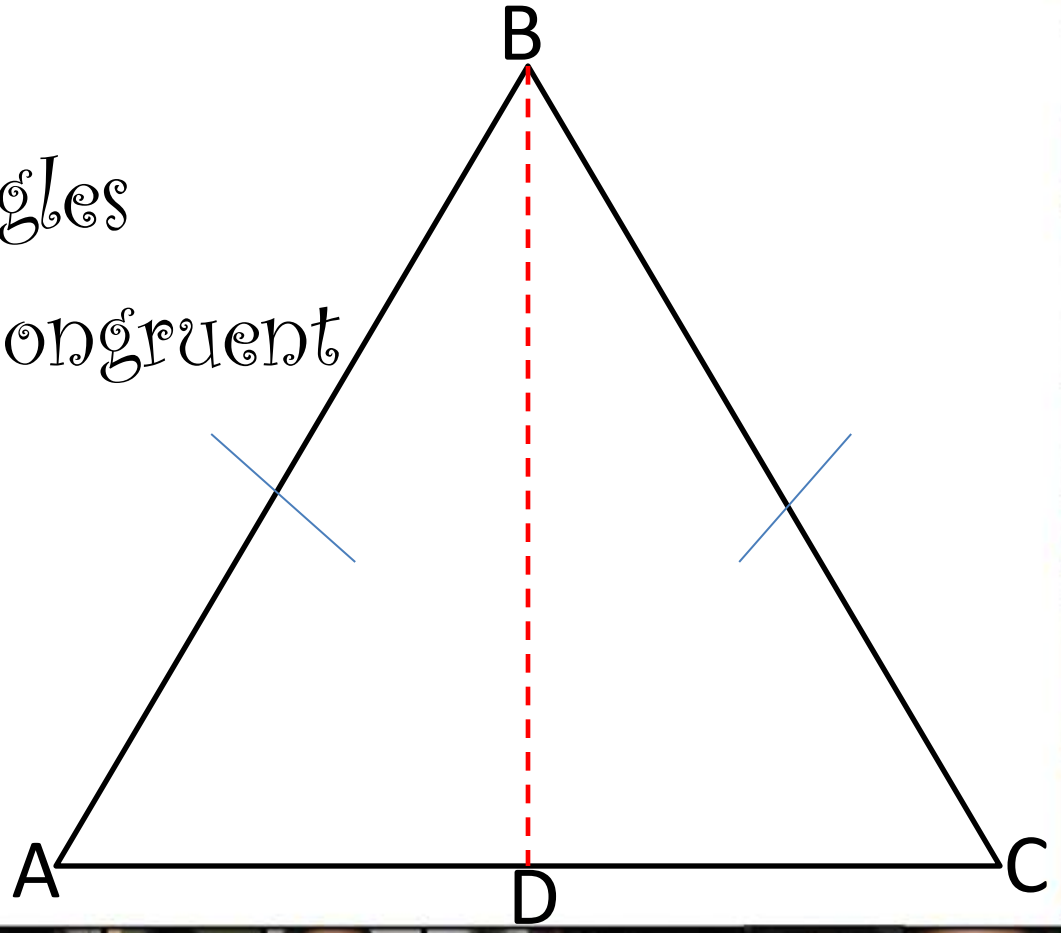


Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Given: Isosceles triangle ABC with legs AB and BC.

Prove that angles A and C are congruent



Steps	Reasons
Isosceles triangle ABC with legs AB and BC.	Given
$\overline{AB} \cong \overline{BC}$	Def of isosceles triangle
Construct ray BD, the angle bisector of angle ABC	Construction
$\angle ABD \cong \angle CBD$	Def. Of angle bisector
$\overline{BD} \cong \overline{BD}$	Reflexive Prop.
$\Delta ABD \cong \Delta CBD$	SAS Post.
$\angle A \cong \angle C$	CPCTC

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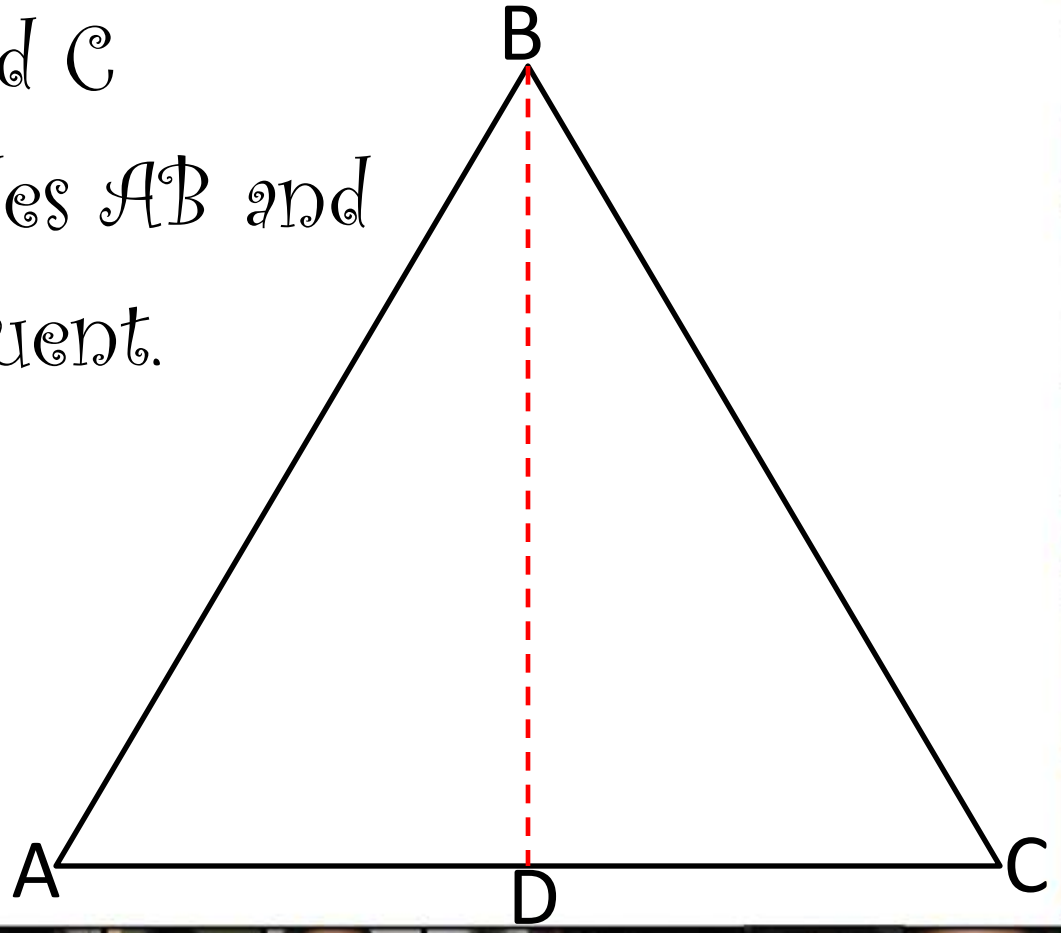
Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite the angles are congruent

Given: Triangle ABC with congruent angles A and C

Prove that sides AB and BC are congruent.



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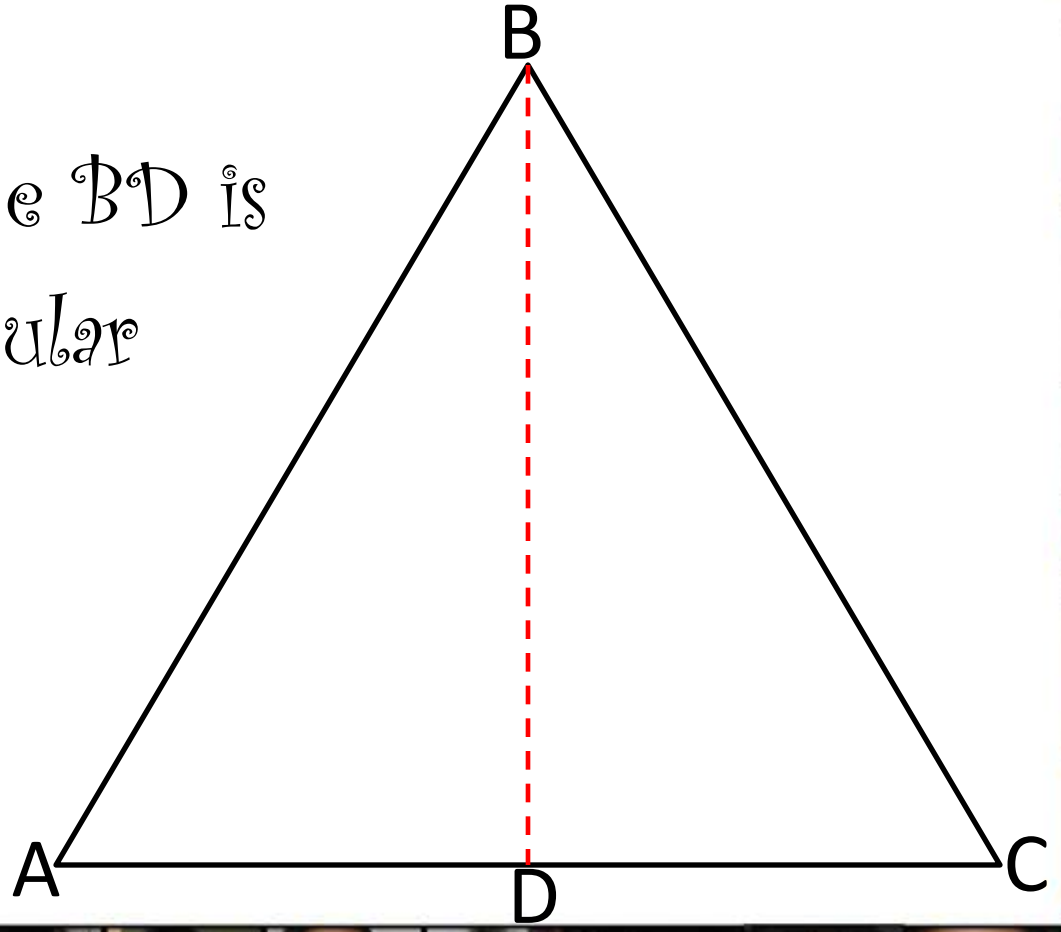
Theorem:

The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

Given: Isosceles triangle ABC with vertex angle B

Prove that line BD is the perpendicular bisector of segment AC.



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$\angle ADB \cong \angle CDB; \overline{AD} \cong \overline{CD}$	CPCTC

Steps	Reasons
$\angle ADB$ and $\angle CDB$ are both right angles	If two angles are congruent supplementary then each of them is a right angle
$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$	Def of a perpendicular bisector

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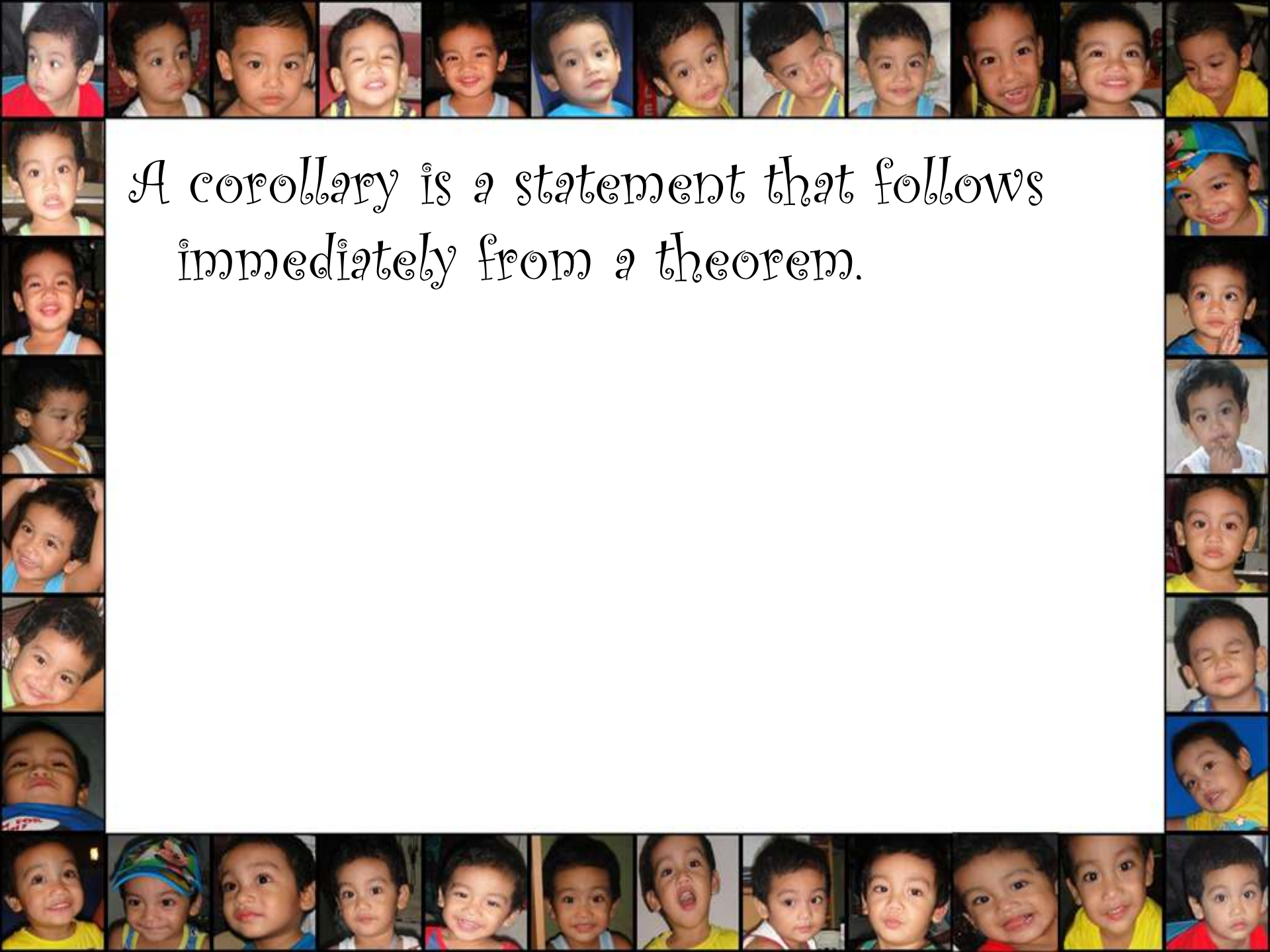
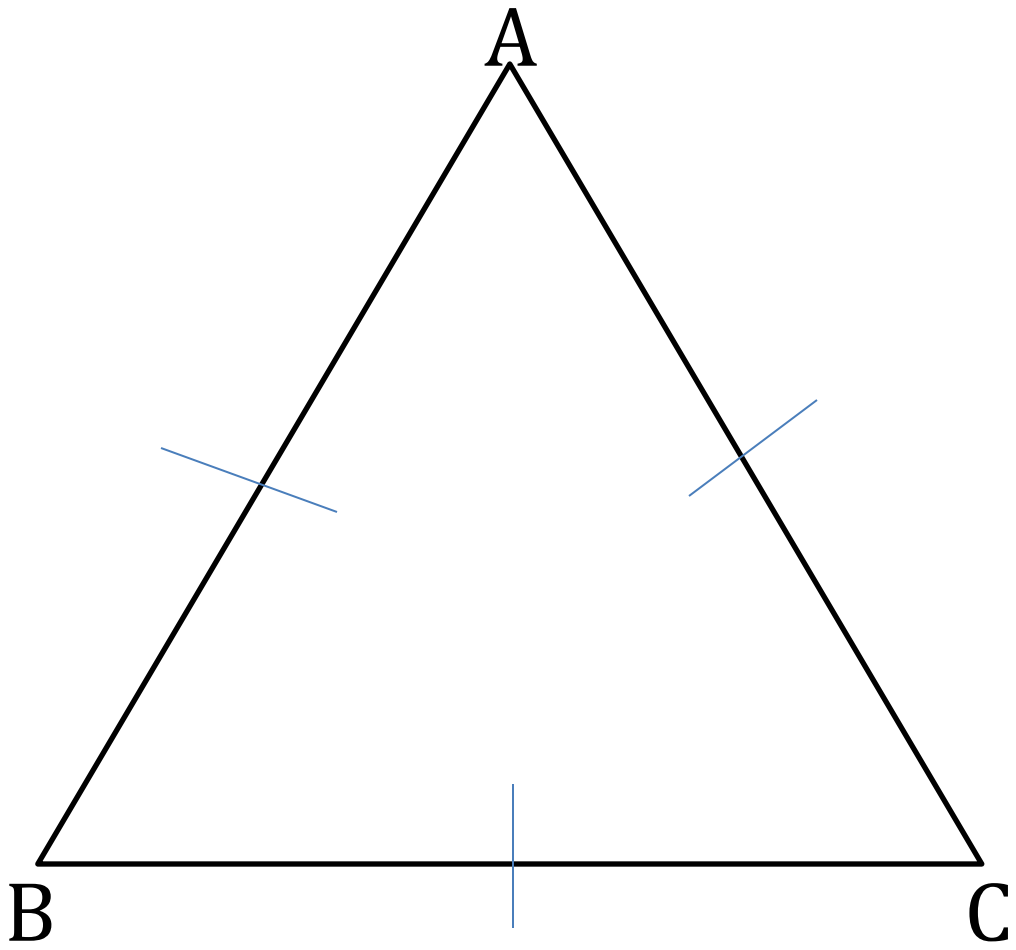
Steps	Reasons
∠ADB and ∠CDB are both right angles	If two angles are congruent supplementary then each of them is a right angle
$\overline{BD}$ is the perpendicular bisector of AC	Def of a perpendicular bisector

A corollary is a statement that follows immediately from a theorem.



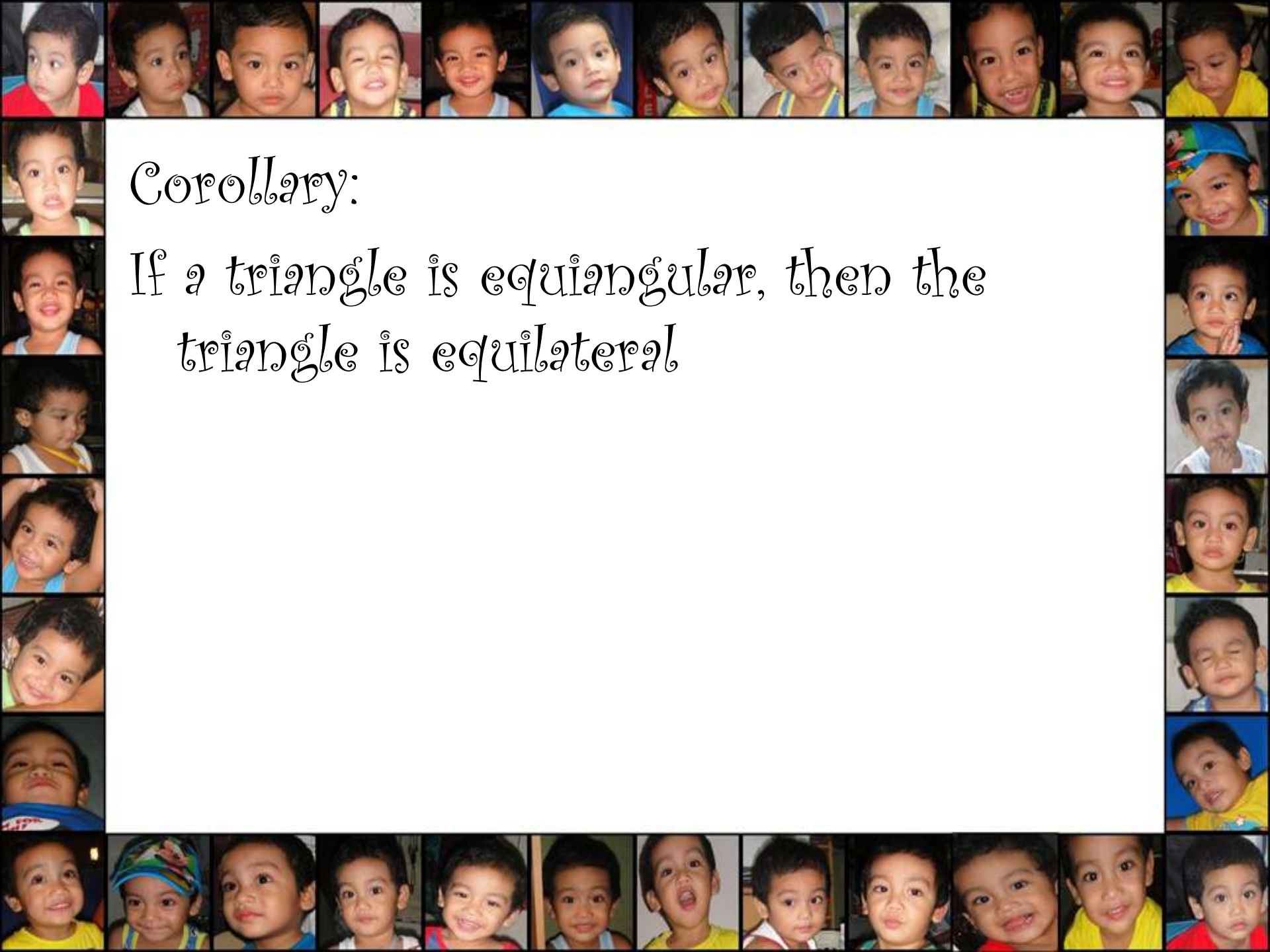
Corollary:

If a triangle is equilateral, then the triangle is equiangular.


A
B
C

Steps	Reasons
Equilateral triangle ABC	Given
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Def of an equilateral triangle
$\angle C \cong \angle A \cong \angle B$	Isosceles Triangle Theorem
Triangle ABC is equiangular	Def. Of an equiangular triangle

Steps	Reasons
Equilateral triangle ABC	Given
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Def of an equilateral triangle
$\angle C \cong \angle A \cong \angle B$	Isosceles Triangle Theorem
Triangle ABC is equiangular	Def. Of an equiangular triangle



Corollary:

If a triangle is equiangular, then the triangle is equilateral

Steps	Reasons
Equiangular triangle ABC	Given
$\angle C \cong \angle A \cong \angle B$	Def. Of an equiangular triangle
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Converse of Isosceles Triangle Theorem
Triangle ABC is equiangular	Def of an equilateral triangle

Steps	Reasons
Equiangular triangle ABC	Given
$\angle C \cong \angle A \cong \angle B$	Def. Of an equiangular triangle
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$	Converse of Isosceles Triangle Theorem
Triangle ABC is equiangular	Def of an equilateral triangle