2-1 Conditional Statements

A ***conditional*** statement is an if-then statement. The part following if is the ***hypothesis*** and the part following the then is the ***conclusion***. Many sentences can be written as conditionals.

A conditional statement can have a truth value of true or false.

Venn diagrams are used to understand true conditional statements.

The ***converse*** of a conditional switches the hypothesis and the conclusion. It is possible for a conditional and its converse to have different truth values.

Symbolic form can be used to represent a conditional and its converse. In symbolic form, the letter $\mathbf{P}$ stands for the hypothesis and the letter $\mathbf{Q}$ stands for the conclusion.

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{P \to Q}$ conditional	$\mathbf{Q \to P}$ converse	$\mathbf{P \Leftrightarrow Q}$ biconditional
T	**F**	**F**	**T**	**F**
T	**T**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**
F	**T**	**T**	**F**	**F**

2-2 Biconditionals and Definitions

When a conditional and its converse are true, we can combine them as a true ***biconditional***. This is a statement you get by connecting the conditional and its converse with the word and. You can write a biconditional more precisely by joining the parts of each conditional with the phrase if and only if.

A good definition is reversible. That means you can write a good definition by a true biconditional.

2-3 Deductive Reasoning

Deductive reasoning (or logical reasoning) is the process of reasoning logically from given statements to a conclusion. If the given statements are true, deductive reasoning produces a true conclusion.

Law of Detachment
If a conditional is true and its hypothesis is true, then its conclusion is true.

In symbolic form:
If $\mathbf{P \to Q}$ is a true statement and $\mathbf{P}$ is true, then $\mathbf{Q}$ is true.

Law of Syllogism

The Law of Syllogism allows us to state a conclusion from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.

In symbolic form:

If $\mathbf{P \to Q}$ and $\mathbf{Q \to R}$ are true statements, then $\mathbf{P \to R}$ is a true statement.

2-4 Reasoning in Algebra

In Geometry, we accept postulates and properties as true. We use deductive reasoning to prove other statements. Some properties we accept as true are the properties of equality from algebra.

Properties of Algebra

1.	Addition Property	If $\mathbf{a = b}$, then $\mathbf{a + c = b + c}$.
2.	Subtraction Property	If $\mathbf{a = b}$, then $\mathbf{a - c = b - c}$.
3.	Multiplication Property	If $\mathbf{a = b}$, then $\mathbf{a \cdot c = b \cdot c}$, for $\mathbf{c \neq 0}$.
4.	Division Property	If $\mathbf{a = b}$, then $\mathbf{\frac{a}{c} = \frac{b}{c}}$, for $\mathbf{c \neq 0}$.
5.	Reflexive Property	$\mathbf{a = a}$
6.	Symmetric Property	If $\mathbf{a = b}$, then $\mathbf{b = a}$.
7.	Transitive Property	If $\mathbf{a = b}$ and $\mathbf{b = c}$, then $\mathbf{a = c}$.
8.	Substitution Property	If $\mathbf{a = b}$, then $\mathbf{b}$ can replace $\mathbf{a}$ in any expression.
9.	Distributive Property	$\mathbf{a(b + c) = ab + ac}$

The Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. We can use properties of congruence to justify statements.

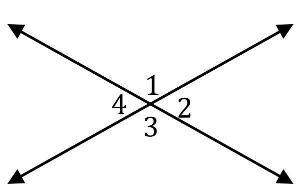
Properties of Congruence

1.	Reflexive property	$\overline{AB} \cong \overline{AB}$; $\angle A \cong \angle A$
2.	Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
3.	Transitive Property	If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$ then $\overline{AB} \cong \overline{EF}$. If and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

2-5 Proving Angles Congruent

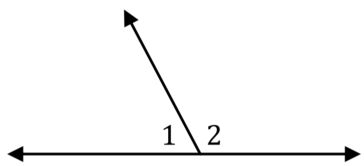
Identifying Angle Pairs

1. Vertical Angles – Two angles whose sides form two pairs of opposite rays.



$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$ are vertical angles

2. Adjacent Angles – two coplanar planes with a common side, a common vertex, and no common interior points.



3. Complementary Angles – two angles whose measure have sum $90°$.
   Each angle is called the complement of the other.

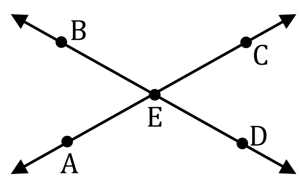
4. Supplementary Angles – two angles whose measure have sum $180°$.
   Each angle is called the supplement of the other.

NOTE: Whether you draw diagram or use a given diagram, we can make some conclusions directly from the diagrams. We can conclude that angles are
1. adjacent angles
2. adjacent supplementary angles
3. vertical angles

Unless there are marks that give the information, we cannot assume:
1. angles or segments are congruent
2. an angle is a right angle
3. lines are parallel or perpendicular

Theorem is a statement that we prove.

Theorem 2-1: ***Vertical Angles Theorem***
Vertical angles are congruent.



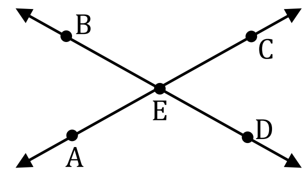
In the figure above, we need to prove that $\angle AEB \cong \angle CED$.

Two-column Proof

Steps	Reasons
1. $\angle AEC$ is a straight angle.	Given
2. $\angle BED$ is a straight angle.	Given
3. $m\angle AEC = 180°$	Definition of a straight angle.
4. $m\angle BED = 180°$	Definition of a straight angle.
5. $m\angle AEB + m\angle BEC = m\angle AEC$	Angle Addition Postulate (AAP)
6. $m\angle BEC + m\angle CED = m\angle BED$	Angle Addition Postulate (AAP)
7. $m\angle AEB + m\angle BEC = 180°$	Substitution
8. $m\angle BEC + m\angle CED = 180°$	Substitution
9. $m\angle AEB + m\angle BEC = m\angle BEC + m\angle CED$	Transitive property/ Substitution

10. $m\angle AEB = m\angle CED$	Subtraction Property (SPE) subtracted $m\angle BEC$ on both sides of the equation)
11. $\angle AEB \cong \angle CED$	Definition of Congruence

Theorem 2-2: ***Congruent Supplements Theorem***



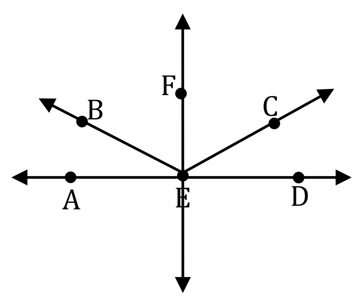
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

In the figure above, we need to prove that $\angle AEB \cong \angle CED$.

Two-column Proof

Steps	Reasons
1. $\angle AEB$ and $\angle BEC$ are adjacent supplementary angles.	Given
2. $\angle BEC$ and $\angle CED$ are adjacent supplementary angles.	Given
3. $m\angle AEB + m\angle BEC = 180°$	Definition of Supplementary Angles
4. $m\angle BEC + m\angle CED = 180°$	Definition of Supplementary Angles
5. $m\angle AEB + m\angle BEC = m\angle BEC + m\angle CED$	Transitive property/ Substitution
6. $m\angle AEB = m\angle CED$	Subtraction Property (SPE) subtracted $m\angle BEC$ on both sides of the equation)
7. $\angle AEB \cong \angle CED$	Definition of Congruence

Theorem 2-2: ***Congruent Complements Theorem***



If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

In the figure above, we need to prove that $\angle AEB \cong \angle CED$.

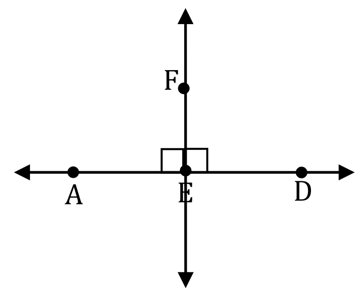
Two-column Proof

Steps	Reasons
1. $\angle AEB$ and $\angle BEF$ are complementary angles.	Given
2. $\angle CED$ and $\angle BEF$ are complementary angles.	Given
3. $m\angle AEB + m\angle BEF = 90°$	Definition of Complementary Angles
4. $m\angle CED + m\angle BEF = 90°$	Definition of Complementary Angles
5. $m\angle AEB + m\angle BEC = m\angle BEC + m\angle CED$	Transitive property/ Substitution
6. $m\angle AEB = m\angle CED$	Subtraction Property (SPE) subtracted $m\angle BEC$ on both sides of the equation)

7. $\angle AEB \cong \angle CED$	Definition of Congruence

Theorem 2-4:
All right angles are congruent.

In the figure on the right, we need to prove that $\angle AEF \cong \angle BEF$

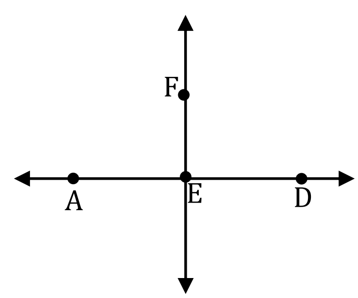


Two-column Proof

Steps	Reasons
1. $\angle AEF$ is a right angle.	Given
2. $\angle DEF$ is a right angle.	Given
3. $m\angle AEF = 90°$	Definition of Right Angles
4. $m\angle DEF = 90°$	Definition of Right Angles
5. $m\angle AEF = m\angle DEF$	Transitive property/ Substitution
6. $\angle AEF \cong \angle BEF$	Definition of Congruence

Theorem 2-5:
If two angles are congruent and supplementary,
Then the two angles are right angles.

In the figure on the right, we need to prove that $\angle AEF$ and $\angle BEF$ are right angles.



Two-column Proof

Steps	Reasons
1. $\angle AEF$ and $\angle DEF$ are congruent.	Given
2. $\angle AEF$ and $\angle DEF$ are supplementary.	Given
3. $m\angle AEF = m\angle DEF$	Definition of Congruent Angles
4. $m\angle AEF + m\angle DEF = 180°$	Definition of Supplementary Angles
5. $m\angle AEF + m\angle AEF = 180°$	Substitution
6. $2m\angle AEF = 180°$	Add
7. $m\angle AEF = 90°$	Division Property (DPE)
8. $m\angle DEF = 90°$	Substitution
9. $\angle AEF$ and $\angle BEF$ are right angles.	Definition of right angles.